JCT Signals Symposium 2026

ZERO-QUEUE TRAFFIC CONTROL USING GREEN-TIMES AND PRICES

by

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Abstract

This paper gives some simple examples to illustrate the capacity-maximising property of traffic signal control policy P_0 and shows that this capacity-maximising property is not possessed by all responsive signal control policies.

The paper also illustrates how an equilibrium consistent with traffic signal control policy P_0 may be enhanced by using prices; it is shown that in a simple network prices may be utilised to drive queues to zero or near zero in the P_0-consistent equilibrium. This illustrates the general result obtained in the recent journal paper (available via White Rose Research Online):

Smith, M.J., Iryo, T., Mounce, R, Satsukawa, K., Watling, D., 2022, Zero-queue traffic control, using green-times and prices together. Transportation Research Part C: Emerging Technologies, 138, 103630.

This paper aims to introduce and illustrate the general results obtained in the above paper.

Keywords: Capacity-maximising traffic signal control; Traffic signal control and pricing.

1. Traffic signal control and route choice; a brief introduction and background

1.1. Traffic signal control

Webster (1958) was one of the first to seek to model signal timings and their effect on traffic flow at a single junction; assuming that average flows are essentially fixed, not changed by signal timing changes. He considered mathematical and simulation models of traffic signal control at a single junction, to design fixed time or time of day signal timings. He proposed the

well-known ‘‘equisaturation’’ policy; at a signal controlled junction with just two conflicting approaches the equisaturation policy chooses greentime proportions which equalise the degrees of saturation of the two conflicting approaches.

This type of rule has often been suggested and utilised in adaptive traffic control systems. Allsop extended Webster’s model to enable calculation of practicable signal timings for a junction of any of a wide range of layouts to be expressed in terms of optimisation subject to linear constraints: this includes maximisation of capacity for known ratios of arrival rates on the various approaches by linear programming (Allsop, 1972) and minimisation of estimated delay for arrival rates within that capacity by convex programming (Allsop, 1971).

This approach was adopted internationally and later for about 20 years in the software package OSCADY (Burrow, 1987). Through the work of his colleagues and of Kimber and Hollis (1979), the requirements for the sequence of stages to be specified and the arrival rates to be within capacity to enable estimated delay to be minimised were relaxed (Allsop, 1992).

Robertson (1969) gives a model of a whole network (TRANSYT) allowing whole network optimisation of traffic signals (for known Origin–Destination (OD) inputs and known route flows). Hunt et al. (1982) developed the real time control system SCOOT; essentially from the TRANSYT model. Heydecker (2004) and Heydecker et al. (2007) propose an adaptive dynamic control system for traffic signals and also consider possible future objectives for traffic signal control. Aboudolas et al. (2009) outline a control designed to optimise a store and forward network model.

1.2. The interaction between traffic signal control and route choice

It is striking that route choices are regarded as fixed in the design of most traffic control strategies; this is true of all the strategies mentioned above in section 1.1.

Wardrop (1952) specified the two basic route-choice principles: selfish routeing or user equilibrium routeing where travellers aim to utilise their own best or quickest routes, and system optimal routeing where the routes are those which minimise total travel time. Allsop (1974) pointed out the importance of allowing for route choices, and other choices, when considering the impacts of signal control changes; quoting Beckmann (1956) who emphasised that travellers are playing their own games while the signal-setter is playing his.

Allsop (1974) specified a network model with coherent traffic signal control variables and route choice variables, seeking to allow the interaction between traffic signal control policies and user-equilibrium, selfish, route choice to be sensibly considered within a single network model.

Gartner (1976) studied the interaction between area traffic control and user equilibrium routeing. Dickson (1981) showed that signal timings which minimise travel time for fixed routes do not necessarily minimise travel time when routeing is variable. Optimisation of signal timings in signal-controlled networks, allowing route choices to vary, is considered by Fisk (1984) and Sheffi and Powell (1983).

There have been many studies since then. Chiou (1999, 2003) consider the interaction between traffic control and route choice, using the TRANSYT model of traffic flow. Van Vuren and Van Vliet (1992) includes the additional consideration of route guidance. Yang and Yagar (1995) and Yang (1996) have considered in detail the interaction between signal control and routeing in saturated road networks. Hu and Mahmassani (1997) have studied (within a model) day to day evolution of network flows under real-time information and reactive signal control.

Smith (1979a, b, 1980), Smith (1987) and Smith and van Vuren (1993) introduce and consider the capacity-maximising policy P_0. This policy is outlined in some detail in section 4.

Daganzo (1998) considers queue spillovers in transportation networks with route choice. Cantarella (2010), Cantarella et al. (1991, 2019) and Cascetta et al (2006) consider signal setting with dynamic process assignment and with deterministic and stochastic equilibrium. Huang et al. (2016) considers repeated anticipatory network traffic control with route choice.

Meneguzzer (1996) gives results of computational experiments with a combined traffic assignment and control model. Meneguzzer (1997) reviews models combining traffic routeing and traffic signal control.

Mounce (2006) gives convergence results in a continuous dynamic queueing model for traffic networks. Mounce (2009) proves existence of equilibrium in a continuous dynamic queueing model for trafffic networks with responsive signal control.

Lammer and Helbing (2008) and Le et al (2015) specify two distributed control systems; the policy in Le et al (2015) is similar to the P_0 policy. Mercader et al (2020) consider distributed max-pressure traffic control based on travel times.

1.3. Traffic control within operational models.

LINSIG (2010) software generates signal timings for given flows; this software is often used in real life for junctions and small networks, and may involve small scale routeing considerations. Schlaich and Haupt (2012) describe a large scale implementation of the equisaturation policy within the VISUM assignment model; the aim is to generate fixed time signal timings over a wide area taking some account of route choice. This was installed as part of the VISUM assignment model by Gentile and Meschini (2013).

2. Traffic signal control and route choice; two simple examples

In this section we consider two simple examples using two very simple control policies on a junction with just two approaches. The two policies are $P_{\mathbf{Fair}}$ and P_0:
Policy $P_{\mathbf{Fair}}$ continually swaps green-time toward the approach with greater bottleneck delay.
Policy P_0 continually swaps green-time toward the approach with greater value of
(saturation flow × bottleneck delay).

2.1. Adding these two signal control policies to a simple network with route choice

2.1.1. The simple network with route choice

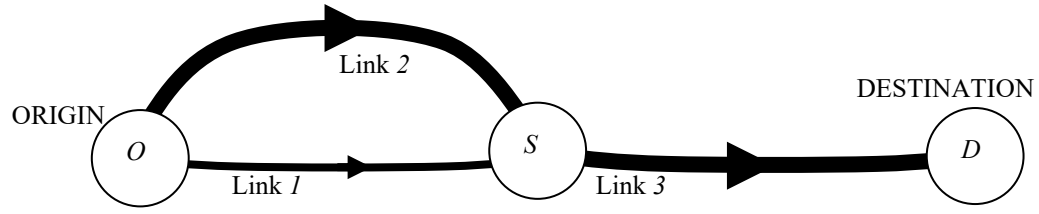


Figure 1. A simple two route signal-controlled network. Two routes (link 1, link 3) and (link 2, link 3) join origin *O* to destination *D*. There is a signal at *S* which allocates green-time to the two links *1* and *2*.

2.1.2. Adding a traffic signal control policy to the figure 1 network with route choice

At the signal-controlled junction *S*: at each time *t*, the traffic signal gives green to either link *1* or link *2* alternately. *The cycle time is fixed at α seconds.*

Suppose that at time zero the signal is green for link *1*. Then as time passes:

the signal switches green from link *1* to link *2* at time αG_{11}; then
the signal switches green from link *2* to link *1* at time $\alpha(G_{11} + G_{12})$; then
the signal switches green from link *1* to link *2* at time $\alpha(G_{11} + G_{12}) + \alpha G_{21}$; then
the signal switches green from link *2* to link *1* at time $\alpha(G_{11} + G_{12}) + \alpha(G_{21} + G_{22})$; then
.
.
.

Here the green-times G_{11}, G_{12}, G_{21}, G_{22} satisfy:

$G_{11} \geq 0$, $G_{12} \geq 0$ and $G_{11} + G_{12} = 1$;
$G_{21} \geq 0$, $G_{22} \geq 0$ and $G_{21} + G_{22} = 1$.

Thus
cycle *1*: the total of the green-times awarded to the *2* links = $\alpha G_{11} + \alpha G_{12} = \alpha$ = the cycle time;
cycle *2*: the total of the green-times awarded to the *2* links = $\alpha G_{21} + \alpha G_{22} = \alpha$ = the cycle time.
And so:

during cycle 1: link 1 is shown green for αG_{11} seconds;
during cycle 1: link 2 is shown green for αG_{12} seconds;
during cycle 2: link 1 is shown green for αG_{21} seconds;
during cycle 2: link 2 is shown green for αG_{22} seconds.

Cycle *2* green-times (G_{21}, G_{22}) are determined by changing the cycle 1 green-times (G_{11}, G_{12}) *by swapping some green-time from the less pressurised link to the more pressurised link.* (1)

Of course we next suppose that cycle 3 green times are determined by changing the cycle 2 green-times (G_{21}, G_{22}) according to (1), cycle 4 green times are determined by changing the cycle 3 green-times (G_{31}, G_{22}) according to (1), cycle 5 green times are determined by changing the cycle 4 green-times (G_{41}, G_{42}) according to (1), and and so on.

All link pressures here, in this section 2, are defined in terms of the *delays* measured or estimated during cycle 1, cycle 2, cycle 3, ... These link pressures are used to select link green-times for cycle 2, cycle 3, cycle 4 and so on.

2.1.3. Fixed network data.

Constant free-flow link 1 travel cost = c_1 (seconds).
Constant free-flow link 2 travel cost = c_2 (seconds).
Constant saturation flow at the exit of link *1* = s_1 (vehicles per second).
Constant saturation flow at the exit of link *2* = s_2 (vehicles per second).
Constant saturation flow at the exit of link *2* = s_3 (vehicles per second).

Assumptions: $c_1 < c_2$ and $s_1 < s_2 < s_3$. (Perhaps link 1 has 1 lane, link 2 has 2 lanes and link 3 has 3 lanes.)
Demand: Here there is supposed to be a steady demand T (vehicles per second) travelling from O to D, where

$$s_1 < T < s_2.$$

2.1.4. Variable network data.

G_1 = proportion of green-time allocated to link 1 during cycle 1 (seconds),
G_2 = proportion of green-time allocated to link 2 during cycle 1 (seconds),
b_1 = average bottleneck delay felt by traffic exiting link *1* during cycle 1 (seconds),

b_2 = average bottleneck delay felt by traffic exiting link *2* during cycle 1 (seconds),
$Press_1$ = Pressure on link *1* during cycle *1*, *as yet undefined*,
$Press_2$ = Pressure on link *2* during cycle *1*, *as yet undefined*.

2.1.5. Definition of two link pressures on links 1 and 2.

In this section we consider just two "link pressures".
Link pressure definition *1*: $Press_1 = b_1$, $Press_2 = b_2$.
Link pressure definition *2*: $Press_1 = s_1 b_1$, $Press_2 = s_2 b_2$.
Following (1) with pressure definition 1 leads to policy $P_{\mathbf{Fair}}$.
Following (1) with pressure definition 2 leads to policy $P_{\mathbf{0}}$.

Any equilibrium consistent with policy $P_{\mathbf{Fair}}$ must equalise the bottleneck delays on links 1 and 2 (otherwise green-times would change); and any equilibrium consistent with policy $P_{\mathbf{0}}$ must equalise the (saturation flow × bottleneck delay) values on links 1 and 2 (otherwise green-times would change).

2.1.6. Definition of average bottleneck delay on links 1 and 2.

Perhaps the simplest definitions of average link-exit bottleneck delays b_1 and b_1 over cycle *1* are:
b_1 = *(average of times taken to exit link 1 in cycle 1) – (free-flow travel time of link 1);*
b_2 = *(average of times taken to exit link 2 in cycle 1) – (free-flow travel time of link 2).*

2.2. Interaction between policy $P_{\mathbf{Fair}}$ and route choice, and policy $P_{\mathbf{0}}$ and route choice.

Here policy *P* will be either policy $P_{\mathbf{Fair}}$ or policy $P_{\mathbf{0}}$.

2.2.1. Equilibrium consistent with control policy *P*

Equilibrium consistent with control policy P will be when
(A) all drivers are satisfied: no driver has an incentive to change route (perhaps because both routes have the same travel time) and
(B) the signal setter is satisfied because the *P* policy holds.

2.2.2. Routeing equilibrium and green-time equilibrium with policies $P_{\mathbf{Fair}}$ and $P_{\mathbf{0}}$.

Equilibrium routeing.
Since $s_1 < T < s_2$, a *routeing equilibrium* is when the two route costs are equal or

$$c_1+b_1 = c_2+b_2. \qquad \text{(ER)}$$

Green-time equilibrium with policy $P_{\mathbf{0}}$:
Green-times G_1, G_2 are in green-time equilibrium under $P_{\mathbf{0}}$ when

$$s_1 b_1 = s_2 b_2. \qquad \text{(EG0)}$$

This is the original $P_{\mathbf{0}}$ policy. If (EG0) does not hold then the green-times will move under $P_{\mathbf{0}}$.

Green-time equilibrium with policy $P_{\mathbf{Fair}}$: green-times G_1, G_2 are in equilibrium following $P_{\mathbf{Fair}}$ when

$$b_1 = b_2. \qquad \text{(EGFair)}$$

If (EGFair) does not hold then the green-times will move under $P_{\mathbf{Fair}}$.

We now consider the two policies P_0 and P_{Fair} separately. We seek delays which are consistent with a P_0 equilibrium and we seek delays which are consistent with a P_{Fair} equilibrium. *These equilibrium delays may be thought of as representing the long-run performance of the policy.*

2.3. Finding equilibrium delays consistent with control policy P_0.

Still $s_1 < T < s_2$. We solve the simultaneous equations (ER) and (EG0). (ER) is the routeing equilibrium condition and (EG0) is the green-time equilibrium condition if the P_0 policy is utilised. So we solve

$$c_1+b_1 = c_2+b_2 \text{ and } s_1b_1 = s_2b_2. \qquad \text{(ER) and (EG0)}$$

Solving these simultaneous equations for the bottleneck delays b_1 and b_2 yields:

$$b_1 = s_2(c_2 - c_1)/(s_2 - s_1) \text{ and}$$
$$b_2 = s_1(c_2 - c_1)/(s_2 - s_1).$$

Equilibrium delays arising from policy P_0 are illustrated in figure 2.

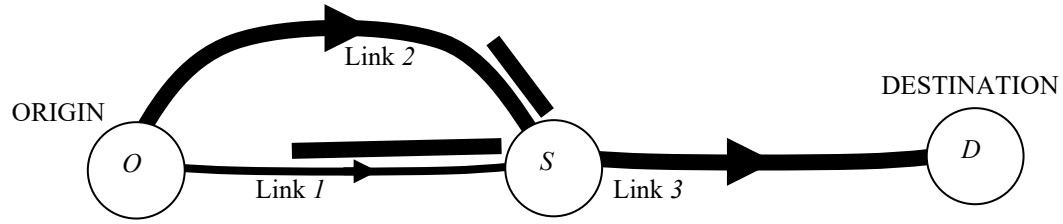


Figure 2. Equilibrium delays if policy P_0 is utilised on the two route network.

2.4. Trying to find the equilibrium delays consistent with control policy P_{Fair}.

Still $s_1 < T < s_2$. We now need to solve the simultaneous equations (ER) and (EGFair). (ER) is the routeing equilibrium condition and (EGFair) is the green-time equilibrium condition if the P_{Fair} policy is utilised. So we now need to solve

$$c_1+b_1 = c_2+b_2 \text{ and } b_1 = b_2. \qquad \text{(ER) and (EGFair)}$$

Solving these simultaneous equations for the bottleneck delays b_1 and b_2 is impossible in this case. So equilibrium delays consistent with policy P_{Fair} do not exist.

It is easy to see this. (ER) and (EGFair) are:

$$c_1+b_1 = c_2+b_2 \text{ and } b_1 = b_2.$$

Thus if (ER) and (EGFair) both hold it follows that

$$b_1 - b_2 = c_2 - c_1 \text{ and } b_1 - b_2 = 0.$$

But one of our assumptions is $c_2 - c_1 > 0$, and so the above conditions are contradictory and so *cannot hold simultaneously.*

This means that under our conditions there is no solution of the two equations (ER) and (EGFair).

2.5. Conclusion of section 2

This section shows that if $s_1 < T < s_2$ then:

(a) with policy P_0, equilibrium bottleneck delays consistent with P_0 exist and
(b) with policy P_{Fair}, equilibrium bottleneck delays consistent with P_{Fair} *do not exist.*

With policy P_{Fair}, a dynamical model has bottleneck delays which grow indefinitely.

3. Traffic signal control, route choice and road pricing

Smith et al. (1994a, b) give many results of network pricing systems; with signal control. Here we study a very simple example.

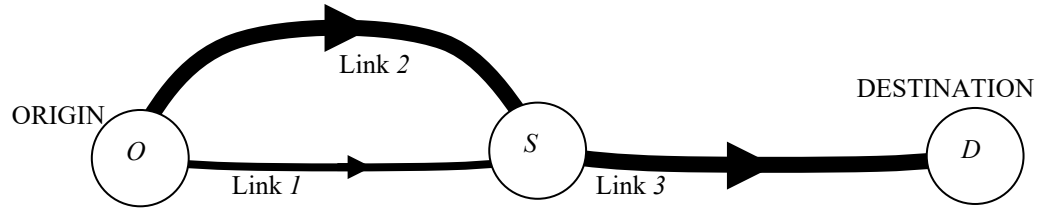


Figure 3. A simple two route signal-controlled network. (Same as figure 1)

On this figure 3 network there are two routes from origin *O* to destination *D* as before: route 1 has link *1* and link *3*, and route 2 has link *2* and link *3.* There is a signal at *S* which allocates green-time to the two links *1* and *2*.

3.1. Fixed network data. This is the same as in section 2.1.3.

3.2. Variable network data.

G_1 = green-time proportion for link 1,
G_2 = green-time proportion for link 2,
b_1 = average bottleneck delay felt by traffic exiting link *1* (seconds),
b_2 = average bottleneck delay felt by traffic exiting link *2* (seconds),
p_1 = price paid by traffic exiting link *1* (seconds),
p_2 = price paid by traffic exiting link *2* (seconds),
$c_1 + b_1 + p_1$ = link 1 travel cost (seconds),
$c_2 + b_2 + p_2$ = link 2 travel cost (seconds),
$Press_1$ = Pressure on link *1, as yet undefined,*
$Press_2$ = Pressure on link *2, as yet undefined.*

3.3. Routeing Equilibrium.

$s_1 < T < s_2$, so at *equilibrium* there are flows on both routes and the route costs are equal or

$$c_1+b_1 + p_1 = c_2+b_2 + p_2.$$

Equilibrium delays consistent with control policy P_0 with zero prices are illustrated in figure 4.

3.3.1. Definition of the link pressures on links 1 and 2 (with prices).

In this section we consider just one form of "link pressure":

***P_0*-with-prices link pressure definitions**: $Press_1 = s_1(b_1 + p_1)$, $Press_2 = s_2(b_2 + p_2)$.

Following (1) with these pressure definitions leads to policy P_0 with prices p_1 and p_2.

3.4. Equilibrium delays consistent with P_0 with prices.

For small prices prices p_1 and p_2, any equilibrium consistent with policy P_0-with-prices must equalise the

(saturation flow × [bottleneck delay + price])

values on links 1 and 2 (otherwise green-times would change).

Equilibrium delays arising from policy P_0 *with zero prices* are illustrated in figure 4.

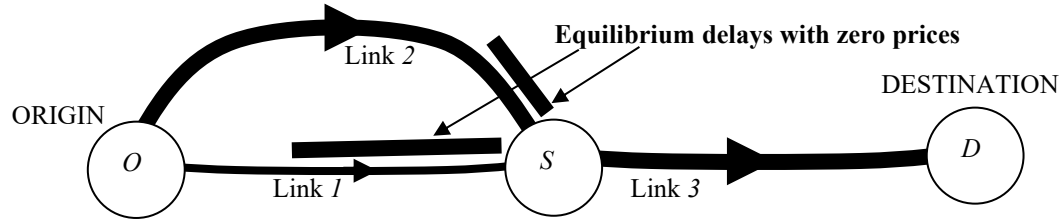


Figure 4. Equilibrium delays if P_0 is utilised with zero prices. This is the same as figure 2.

Now consider slowly raising the prices in figure 4 from zero; under natural conditions the equilibrium delays shown in figure 4 above will slowly shrink. In a simple model delays go to zero. An intermediate result is shown in figure 5 and the zero-queue result with larger prices is illustrated in figure 6. *Here delays have been made zero; replaced by equivalent prices.*

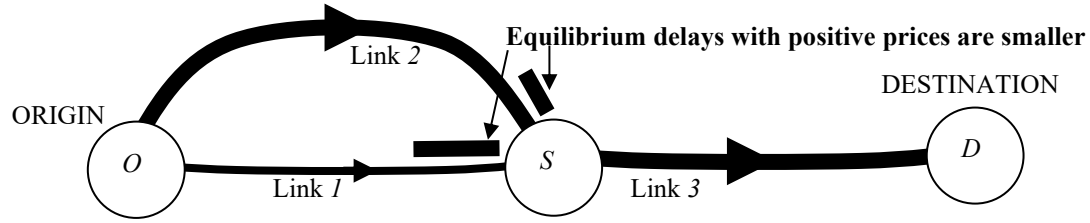


Figure 5. Equilibrium delays are reduced if P_0 is utilized with small positive prices.

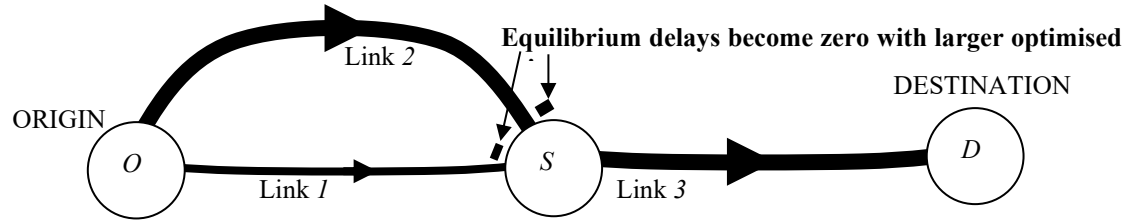


Figure 6. Under suitable conditions equilibrium delays are both zero if prices are increased sensibly, to replace delays.

3.5. Conclusion of section 3

This section illustrates how an equilibrium consistent with policy P_0 may be enhanced by using prices; prices are here used to drive queues to zero or near zero in the P_0-consistent routeing equilibrium. This illustrates the general result obtained in Smith et al. (2022).

4. The P_0 responsive traffic signal control policy; a brief description

Summary. This section 4 briefly describes the traffic signal control policy P_0 and some related policies. Policy P_0 was designed *to maximise network capacity* under certain conditions and it has been shown, in Smith (1979a, b, 1980), Smith et al. (2019a, b, 2022) and elsewhere, that the P_0 policy and related policies do indeed maximise the capacity of many networks under various conditions. These conditions allow for travellers to swap to quicker or cheaper routes, so these policies allow reasonably for drivers' future route choices. *This is very different to usual traffic control policies which do not seek to allow sensibly for route choices.*

4.1. Introduction

Very many researchers have sought to model capacity-constrained traffic assignment (traffic routeing), aiming to achieve more realistic or tractable routeing models with a better representation of likely route choices and the likely size (in number of vehicles) or the spatial extent (in metres) of queues. Very many papers have been written on this subject. The basic routeing assumptions here broadly follow Wardrop (1952), who suggested that vehicle routeing will likely tend toward an *equilibrium* distribution of flows where no driver has any incentive to change route.

Smith (1979a, b, 1980), Van Vuren and Van Vliet (1992) and Smith and van Vuren (1993) consider such traffic equilibrium routeing with responsive traffic control; and introduce the P_0 control policy. Smith (2010) suggests a method for calculating fixed-time signal green-times taking routeing into account.

4.1.1. Adding signal control policies to a network with route choice

All the control policies considered here use signal stages and stage pressures; all the control policies we consider in this paper are "pressure driven" where stage pressures are specified for every stage; and then, as time passes, at each junction, stage green-times are continually

swapped away from stages with a lower pressure toward stages with a higher pressure. (2)

The swapping (2) occurs only between one cycle and the next cycle. The aim is to approximately equalise the pressures on all stages at each junction; while respecting constraints on stage green-times.

Different responsive signal control policies arise by choosing different definitions of stage pressure.

Suppose given a capacitated network $[N, L]$, with a set of nodes N and a set of links L. In this paper we suppose that each link represents a single traffic lane with a single exit saturation flow.

We consider the simplest of real-life responsive traffic control systems where at each junction:
(i) the cycle time is given and fixed; and
(ii) there is a given ordered collection of fixed sets of non-conflicting approaches to be given green in order.

4.1.2. Compatible sets of approaches to a junction

At each signal-controlled junction: at each time t, a traffic signal gives green to a a set of non-conflicting approaches while all other approach lanes at the same junction are shown red. The period of time t during which a set of non-conflicting approaches is given green is in this paper called a "stage". As time passes the signal switches green from one set of non-conflicting approaches to the next set of non-conflicting approaches. All-red is permitted: in this case the set of lanes given green is empty.

We assume that there are n stages. A_j will denote an arbitrary *compatible set of approach links,* where $1 \leq j \leq n$. All approaches in the the set A_j will be shown green during stage j.

We insist here that the cycle time is to be fixed at (say) α seconds and that there is a given (ordered) sequence

$$[A_1, A_2, \ldots, A_n]$$

of compatible sets of approaches given green during n contiguous time-periods called stages:

$$Stage_1, Stage_2, \ldots, Stage_n.$$

Given that the cycle time is fixed at α seconds, here the proportion of the cycle giving green to A_j [that is (the duration of stage j)/α] will be denoted by G_j; so that the proportions of the cycle giving green to the above n compatible sets of approaches, $A_1, A_2, \ldots, A_n$, are:

$$\{G_1, G_2, \ldots G_{n-1}, G_n\}.$$

Here

$$G_1 \geq 0,\ G_2 \geq 0,\ \ldots G_{n-1} \geq 0,\ G_n \geq 0 \text{ and } G_1 + G_2 + \ldots + G_{n-1} + G_n = 1.$$

Thus

the total of the green-times awarded to the n stages $= \alpha G_1 + \alpha G_2 + \ldots + \alpha G_{n-1} + \alpha G_n$
$= \alpha$ = the cycle time.

The set $\{G_1, G_2, \ldots G_{n-1}, G_n\}$ of stage green-time proportions must satisfy further linear constraints, to ensure

(i) that each link has a minimum green-time in each cycle and
(ii) that clearance times are respected.

See Allsop (1971) for a picture of all such constraints for a particular junction.

At each junction and for each cycle, the proportions of green-time awarded to the stages during the cycle will here be determined at the start of the cycle, using the green-time proportions and stage pressures experienced during the previous cycle. This is to be done here:

by swapping some green-time from less pressurised stages to more pressurised stages. (3)

Stage pressures in (3) will be defined in terms of flows, queues and delays measured during the previous cycle.

The stage j pressure may be thought of as the stress that stage j (or A_j) under; so it makes sense to swap green-time away from stages with a low pressure (or stress) toward stages with a higher pressure (or stress). Such a control system is said to be "pressure-driven".

We now explain simply how stage pressures may be defined and utilised within a control system following P_0.

We suppose that flows exiting from different links in the same set A_j do not conflict or interact. For a given junction and a given cycle we let:

α = the cycle time (seconds, supposed fixed here),
s_i = saturation flow at the link i exit (vehicles/second, supposed fixed),
b_i = average bottleneck delay felt by traffic exiting link i during the previous signal cycle (seconds),
G_j = proportion of the previous cycle in stage j, giving green to A_j (unit free),
x_i = average flow out of the link i exit during the previous cycle (vehicles/second) and
Q_i = average queue size in the queue at the exit of link i during the previous cycle (number of vehicles).
$Press_j$ = Pressure on stage j = pressure on A_j or $A(j)$.

4.1.3. Definition of average bottleneck delay

Perhaps the simplest definition of average link-exit bottleneck delay b_i over a cycle is:

b_i = *(average of times taken to exit link i in the cycle) – (free-flow travel time of link i).*

4.1.4. Stage pressures yielding the equi-saturation control policy

Probably the earliest example of a pressure-driven traffic signal control policy is the equisaturation policy specified by Webster (1958). In this policy the pressure on stage j equals the maximum of the degrees of saturation $x_i/(s_iG_j)$ over the links i given green during stage j; thus

$$Press_j = \max\{x_i/(s_iG_j);\ \text{link } i \text{ belongs to } A_j\}$$

Webster proposed that stage green-times should be chosen so that all stage pressures at a single signal-controlled junction are equal, or as equal as possible. (Hence his famous proposed "equi-saturation" signal control policy.)

Webster regarded this simple rule as a reasonable approximation to the policy of minimising the total rate of delay experienced at the junction, assuming that traffic flows are fixed, unaltered if the signal timings are changed. Webster suggested equisaturation as a way of selecting stage green-times for implementation over a fairly long period of time, the morning peak or the inter-peak for example. So his equisaturation rule for setting signals was aimed at choosing fixed time signal settings; however "equisaturation" is often utilised as part of modern responsive traffic signal control systems.

4.1.5. Stage pressures yielding the P_0 control policy and related policies

Smith et al. (1979a, b, 1980) introduce the stage j pressures:

$$Press_j = \sum_{\text{link } i \text{ belongs to } Aj} s_i b_i.$$

These stage j pressures lead to the P_0 control policy. A dynamical version of this policy (stated in Smith et al. (2022) and elsewhere) swaps green-time from less pressurised stages to more pressurised stages at each junction, aiming to equalise stage pressures at each junction. These swaps occur between one cycle and the next.

Smith and van Vuren (1993) and Smith et al. (2019a, b) introduce, for a constant vector $\mathbf{h}$, policy $P_{\mathbf{h}}$, a biased form of policy P_0. This policy utilizes the stage j pressures

$$Press_j = \sum_{\text{link } i \text{ belongs to } Aj} (s_i b_i + h_i),$$

again swapping green-time from less to more pressurised stages.

4.2. Past P_0 studies

Smith (1979a, b, 1980) show that, under certain conditions, an equilibrium consistent with the original P_0 policy certainly exists if demand is within network capacity. (We say that P_0 maximises the capacity of the network.)

Smith (1987) extends this result to allow wider conditions.

Smith et al. (2019a) extends the above capacity-maximising results to a new policy: a biased form of policy P_0 called $P_{\mathbf{h}}$. Here the bias vector $\mathbf{h}$ represents the estimated attractiveness of alternative routes, and helps $P_{\mathbf{h}}$ reduce unnecessary queueing delays.

Smith et al. (2022) extends these results further to include prices (as well as delays) and shows that a pricing variant of P_0 does, under certain conditions, eliminate queues at equilibrium.

4.3. An explanation of the capacity-maximising property of P_0

The capacity maximising property of P_0 is as follows. Suppose given a capacitated signal-controlled network model which allows route choice. Suppose also that there is a given inelastic demand D which can be met by some (typically unknown) signal timings and some (typically unknown) route-flows; then under additional conditions there is a

[route-flow pattern and signal timing]

pair for which

(a) the routeing satisfies Wardrop's equilibrium condition; and

(b) the signal timings satisfy P_0.

This is not true for most traffic signal control policies.

The above capacity-maximisation condition holds under various different "additional conditions". Yet the policy requires no knowledge of the origin-destination distribution and so may be implemented locally and automatically.

We believe that these properties of P_0 make the policy, and the related policies P_h, ideal for the digital age and so likely to be important. There are many opportunities for development of these policies, especially using pricing and signal control together (see Smith et al. (2022)).

4.4. Conclusion to section 4

This section has briefly described the traffic signal control policy P_0. This policy was designed to maximise network capacity under certain conditions and it has been shown, in Smith (1979a,b, 1980), Smith et al. (2019a, b, 2022) and elsewhere, that the P_0 policy and related policies do indeed maximise the capacity of many networks under various conditions.

5. Traffic signal control and pricing results on a model of the City of York.

In this section we give some results obtained by Dr Mustapha Ghali using the CONTRAM dynamic simulation model. CONTRAM was designed and implemented by TRL.

The user guide to CONTRAM version 4 (1982) by Leonard and Gower states that CONTRAM was designed:

" . . . . as a traffic assignment model for use in the design of traffic management schemes. The model predicts vehicle routes, flows and queues in a network of streets and junctions; junctions may be controlled by traffic signals or 'give-way' rules. It is assumed that the numbers of trips between each origin and destination are known and that they may vary with time so that the growth and decay of congestion in peak periods can be studied. Allowance is made for the physical size of queues which may block back and restrict the throughput capacity at upstream junctions. Up to three classes of vehicles (eg cars, buses and lorries) can be represented, and selected vehicles (usually buses) can be sent along fixed routes. CONTRAM provides comprehensive information on traffic conditions, including link delay times, turning movements and fuel consumption, to help the traffic engineer to understand and assess the merits of alternative traffic management schemes."

See also Taylor (2003).

Our results here demonstrate that, compared to two other signal control policies, the P_0 control policy with prices as illustrated in the simple examples in section 2 above deliver much the best performance in terms of both total queueing delay and total travel time. *Especially noteworthy*

is the near 60% reduction in total queueing time achieved by using the P_0 policy; with rather small total of charges paid.

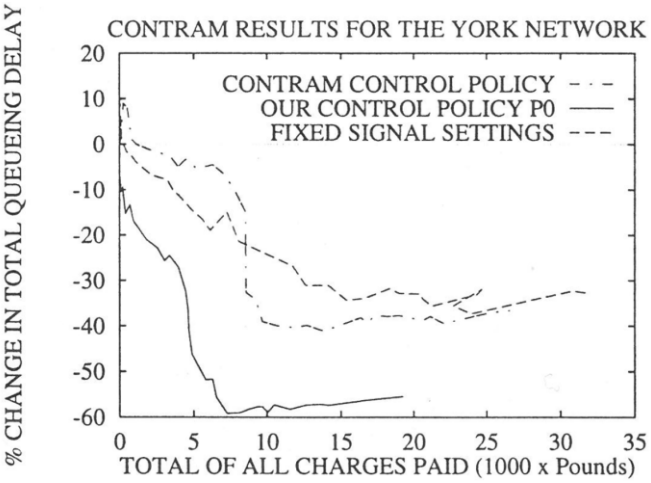


Figure 7. The solid curve gives the change in total queueing time achieved by using the P_0 policy with delay-based pricing, as illustrated in figures 4, 5 and 6 in section 3 above. In this figure the results of using fixed-time signals and the CONTRAM control policy (still with delay-based pricing) are also shown (as dashed curves). Clearly P_0 achieves much the best results at much lower charges.

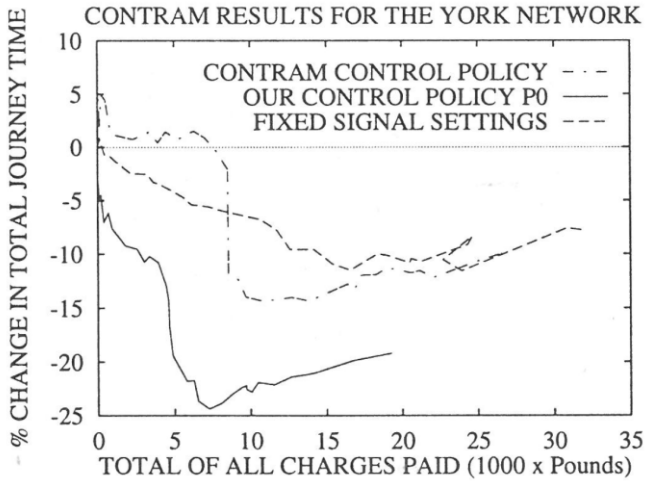


Figure 8. The solid curve gives the change in total journey time achieved by switching to the P_0 policy; with delay-based pricing as illustrated in figures 4, 5 and 6 in section 3 above. In this figure the results of using fixed-time signals and the CONTRAM control policy (with delay-based pricing) are also shown (as dashed curves). Clearly P_0 achieves much the best results at much lower charges.

6. Conclusion

This paper gives some simple examples to illustrate the capacity-maximising property of control policy P_0 and shows that this capacity-maximising property is not possessed by all responsive signal control policies; by showing that it is not possessed by $P_{\mathbf{Fair}}$.

The paper has also illustrated how an equilibrium consistent with policy P_0 may be enhanced by using prices; prices are here used to drive queues to zero or almost zero in the P_0-consistent equilibrium. This illustrates the general ideal result obtained in Smith et al. (2022).

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